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ON PROOFS OF THE EXISTENCE OF NON-COMPOSITE POLYHEDRA WITH RHOMBIC VERTICES

V. I. Subbotin[1]

[1] Platov South Russian State Polytechnic University (NPI),
132 Prosveshheniya Ave., Novocherkassk 346400, Russia

E-mail: geometry@mail.ru

Dedicated to the 85th anniversary of E. M. Semenov

Abstract. Previously, the author considered closed convex polyhedra in E^3 with symmetrical rhombic vertices and regular faces, the so-called *RR*-polyhedra, in two cases: 1) regular faces of such polyhedra are of the same type, 2) regular faces are of different types, but the polyhedra are composite. From the author's previously published works, it follows that in both cases the list of *RR*-polyhedra is complete. In the first case, there were twenty-four of them, and in the second, fifty-four. A vertex V of a polyhedron is called rhombic if the vertex star $\mathrm{Star}(V)$ consists of n equal and identically located rhombi (not squares) having a common vertex V. If the vertex V lies on the rotation axis of order n of the star $\mathrm{Star}(V)$, then V is called symmetric. A symmetric rhombic vertex V is called obtuse-angled if the rhombi of the star $\mathrm{Star}(V)$ at the vertex V meet at their obtuse angles. The star $\mathrm{Star}(V)$ is then called symmetric rhombic. The faces of an *RR*-polyhedron can be divided into two non-empty disjoint sets: the set of equal symmetric rhombic stars that do not have common edges, and the set of regular faces. An example of an *RR*-polyhedron is an elongated rhombododecahedron. If an *RR*-polyhedron has regular faces of one type, then such an *RR*-polyhedron is classified as the first type. Some regular-faced polyhedra with conditional edges, such as the extended rhombododecahedron and the doubly complemented cuboctahedron, are special cases of *RR*-polyhedrons. In this case, the rhombic faces of regular-faced polyhedra with conditional edges, unlike the rhombi of *RR*-polyhedra, are composed only of regular triangles. In this paper, constructive proofs are given for the existence of four non-composite *RR*-polyhedra. For two of them the author previously found the angles of the rhombi that form symmetrical faceted stars of the vertices. In the case of the 25-sided polyhedron, the rhombic vertex has an acute-angled faceted star, i. e. the rhombi in it meet at their acute angles. And in the case of a 31-sided polyhedron the faceted star is obtuse. The existence of two other non-composite *RR*-polyhedra is proven on the basis of the regular dodecahedron and the cube.

Keywords: non-composite RR-polyhedron, faceted vertex star, symmetrical rhombic vertex.

AMS Subject Classification: 52B15.

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1. Introduction

If we disregard the class of regular (platonic) polyhedra and the class of semiregular (archimedean) polyhedra, see [1, 2], it is known that in addition to the infinite series of right prisms and antiprisms, in E^3 there are only ninety-two convex regular-faced polyhedra. They were found empirically by Johnson [3]. The completeness of the list was proven in [4] by V. A. Zalgaller.

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Later, this topic developed along the path of listing all regular-faced polyhedra with *conditional* edges. In this case, an edge is considered conditional if the dihedral angle with this edge is equal to π. Rhombic faces in regular-faced polyhedra with conditional edges are composed of two regular triangles. The final list of all polyhedra with regular faces, which allow conditional edges, can be found, for example, in [5].

In [6] a class of closed convex polyhedra in E^3 (the so-called RR-polyhedra) was considered, some of whose faces are not regular, but rhombic, forming rhombic stars of some vertices; however, the rhombuses are not composed of two regular triangles. In the same place in [6] the existence and uniqueness of two RR-polyhedra were proved, and in [7–9] — a list of this class was found, consisting of twenty-three polyhedra, under the condition of uniformity of regular faces. In [10] the author proved the existence of a twenty-fourth polyhedron, previously omitted in [7–9], with four obtuse-angled rhombic vertices and uniform regular faces, that is, a polyhedron of the first type. Thus, the complete list of RR-polyhedra of the first type consists of twenty-four polyhedra.

In [11] all fifty-four *composite* RR-polyhedra with regular faces of various types were found. In the present paper, proofs are given of the existence of four *non-composite* RR-polyhedra with regular faces of various types, *having no conditional edges*. Earlier in [12], the author found the "characteristic equations" of two of these four polyhedra and, using the characteristic equations, found the angles of the rhombi of the rhombic vertices. Thus, if we add the four RR-polyhedra without conditional edges found in the present paper to the non-composite polytopes with rhombic vertices and conditional edges from [5], we obtain a list of twenty-five non-composite RR-polyhedra of the second type.

2. Basic Concepts

By *bending* of some polyhedral surface M with boundary throughout this paper we mean such an isometric transformation of it that only the dihedral angles of the surface M can change, while the faces as flat polygons remain unchanged. It is also assumed that the symmetry group of the surface M remains unchanged under bending. For brevity, such bendings will sometimes be called *S-bendings*.

A closed convex polyhedron in E^3 is called a *RR-polyhedron* (from the words "rhomb" and "regular") if the set of faces of the polyhedron is the union of two disjoint non-empty sets: the set of faces (rhombs, not squares) that form stars of symmetric rhombic vertices and the set of regular faces. If the said regular faces are of different kinds, then the polyhedron is called a RR-polyhedron of *the second type*; if the regular faces are of the same kind, then it is called a RR-polyhedron of *the first type*.

By the star $\mathrm{Star}(V)$ of the vertex V of a polyhedron we mean the set of all faces that have one common vertex V. If $\mathrm{Star}(V)$ consists of n equal and equally spaced, i. e. converging at the vertex V either with their acute or obtuse angles, rhombuses, then the vertex V is called *n-rhombic*. If V is located on the rotation axis of order n of the set $\mathrm{Star}(V)$, then V is called a *symmetric n-rhombic vertex*.

A rhombic vertex is called *obtuse-angled* if the rhombuses converge at the vertex V with their obtuse angles.

The *free angles* of a rhombic star are the angles with vertices that coincide with the common vertex of two obtuse angles of adjacent rhombuses. If we place regular triangular faces in the free corners of an n-rhombic star, we obtain a polyhedral surface — *rhombic cap*. We will call a *rhombic pyramid* a convex polyhedron obtained by gluing a regular n-gon to the boundary of the cap.

Composite is an RR-polyhedron of the second type that can be divided by planes into parts that are rhombic pyramids and convex polyhedrons with regular faces. Obviously, an RR-polyhedron with an obtuse-angled rhombic vertex V is non-composite, since a rhombic star in this case can consist only of three rhombi that have a common vertex V.

3. On the Existence of an RR-Polyhedron Associated with a Triangular Hebesphenorotunda

Theorem 1. *There is a non-composite RR-polyhedron of the second type with a symmetric obtuse-angled rhombic vertex and twenty-eight regular faces.*

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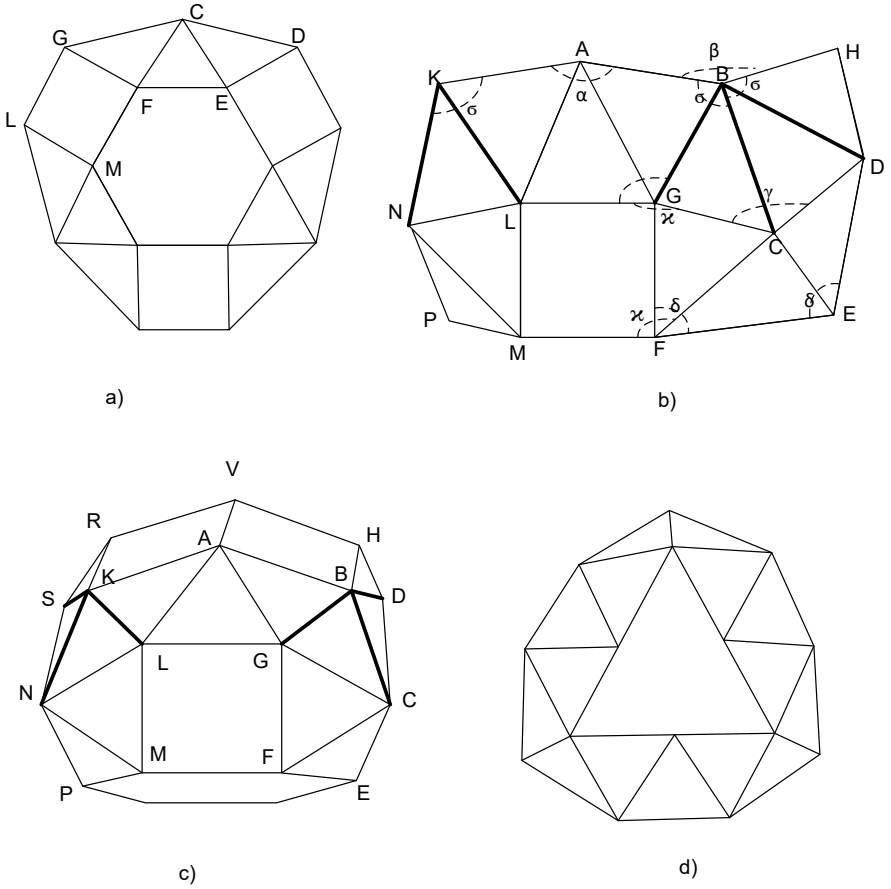

Fig. 1. RR-polyhedron associated with a triangular hebesphenorotunda.

Consider a polyhedral surface (let us denote it by Φ) with an edge that is part of a triangular hebesphenorotunda — the regular-faced Johnson polyhedron J92. The surface Φ, consisting of thirteen faces, is shown in Fig. 1 (a). We will consider the bending of this surface, in which the faces of the surface remain rigid when the dihedral angles change. In addition, we will assume that the symmetry of the surface as a whole is preserved, that is, the rotation-axis L of the third order, passing through the center of the 6-gonal face and perpendicular to it, and the six planes of symmetry passing through L are preserved. Let the dihedral angles of the square faces with the 6-gonal face increase during this S-bending. Then, by virtue of the well-known Cauchy lemma applied to 4-sided angles with vertices F and E

and equivalent to them, the dihedral angles between each two triangular faces increase. Let us consider one of the vertices, for example, C.

As is known, if during deformation of a convex polyhedral angle, preserving its convexity and the invariance of its faces except for one face (let us denote it f), the dihedral angles between the invariant faces do not decrease and at least one of them increases, then the plane angle of face f increases. Therefore, the plane angle GCD, which we denote by γ, increases. If the symmetry of the polyhedral surface is preserved, of course, the same is true for the vertices equivalent to C. In what follows, we will not specifically stipulate that some statements (due to the preservation of symmetry under bending) remain valid for vertices and angles equivalent with respect to the rotation-axis.

Thus, a pair of regular triangles GBC and BCD with a common edge BC can be glued to the plane angle GCD, Fig. 1 b), c). In this gluing, the dihedral angle with the edge BC is chosen to be less than 180°. As a result, we obtain the surface of a 5-gonal pyramid with vertex C. Let δ denote the equal angles GFE and DEF. Let $\delta = \gamma = 108°$. Then the 5-gonal pyramid $CGBDEF$ with vertex C will be regular.

As a result of these constructions, we obtain a polyhedral surface Φ with three pairs of regular triangles glued to it. Let us denote this surface Φ'.

Let us draw a plane p perpendicular to one of the square faces of the surface Φ' so that p passes through the vertices C and N of the regular 5-sided pyramids. Let us reflect symmetrically half of the 6-sided face relative to the plane p. We obtain a flat equilateral trapezoid $KLGB$. Having carried out a similar construction for all three squares, we obtain a surface Φ'', the edge of which is a regular triangle. In Fig. 1 d) the surface Φ'' is schematically depicted from the side of its boundary.

Let us denote the plane angles at the vertices A and B of the surface Φ'', respectively, α and β. Obviously, $\alpha = 180°$, $\beta = 60°$. Our goal will now be to show that there exists a symmetry-preserving bending of the surface Φ'' such that with an increase in β the angle α decreases so that the equality $\alpha = \beta$ is achieved. The last equality is necessary and sufficient for a symmetric obtuse-angled trihedral rhombic cap with vertex V to be glued to the boundary of the surface obtained by bending Φ'', Fig. 1 c).

Let us return to the surface Φ'. When changing the dihedral angles of square faces with a hexagonal one, according to the Cauchy lemma, at the vertices, for example, F and C, there must be four changes of sign. In this case, as usual, increasing and decreasing dihedral angles at the vertices are marked, respectively, with different signs "plus" and "minus". When increasing the dihedral angles of square faces with a hexagonal one, the dihedral angle with the edge FC increases, the dihedral angle with the edge CG decreases, and the dihedral angle with the edge CB, like the plane angle γ, increases. Since both dihedral angles with the edges GL and GC decrease, the plane angle LGB decreases. Recall that, due to the symmetry of the bending, the plane angle KLG and the other two pairs of angles, equivalent to the pair of angles LGB and KLG, also decrease. Therefore, instead of the equilateral trapezoid $KLGB$, we can glue a symmetrical figure $KLGB$ of three regular triangles, in which the dihedral angles at the edges LA and GA are equal to each other and less than the straight ones. Note that with this increase in the angle γ, the dihedral angles at the edges GB and DB increase, which follows from the Cauchy lemma applied to the vertices G and D. Thus, all the dihedral angles at the vertex B increase and, therefore, with an increase in γ, the angle β increases. And since the dihedral angles at the vertex A decrease, the plane angle α decreases.

Thus, as the angles γ increase, the angles β increase, and the angles α decrease. From the above it is obvious that $\max \alpha = 180°$, $\min \beta = 60°$, $\min \gamma = 108°$. To find the possible limits of variation of these angles, we need to find $\min \alpha$, $\max \beta$, $\max \gamma$.

Earlier in [12] the main equation (equation (27) in [12]) for the polyhedron under consideration was obtained:

$$\frac{1+4\cos\lambda-4\cos^2\lambda}{2} = \cos^2\sigma+\sin^2\sigma\cos\left(\arccos\left(\frac{4\cos\gamma-1}{3}\right)-2\arccos\left(\frac{1-\cos\sigma}{\sqrt{3}\sin\sigma}\right)\right). \qquad (1)$$

In this equation λ is the obtuse angle GBL of the isosceles trapezoid $KBGL$, σ denotes the plane angle ABC and the angle CBH, which is equal to it by virtue of symmetry. In this case, λ and σ are functions of γ.

In [12], using computer calculations, the only root of the equation $\gamma \approx 2.01$ was found, or, in degrees, $\approx 115.31^\circ$. The obtuse angles of the rhombuses with the found γ at the vertex V: $\alpha = \beta \approx 112.17^\circ$.

By virtue of the previous reasoning, the angle σ increases with increasing γ. Let us find the value of γ at which the angle σ becomes equal to 120°, and the corresponding σ dihedral angle becomes straight. To do this, obviously, the right-hand side of equation (1) should be equated to $-\frac{1}{2}$. A computer solution of the resulting equation yields the value $\gamma \approx 116.54^\circ$. This means that γ has a limit value of $116.54^\circ\ldots$, at which conditional edges appear on the polyhedral surface Φ''. Thus, $\max\gamma \approx 116,54^\circ$.

Since the left and right sides of equation (1) are $cos\,\alpha$ and $cos\,\beta$, respectively, then substituting $\max\gamma = 116,54^\circ$ into the left and right sides of equation (1), we calculate the values $\min\alpha \approx 104,48^\circ$, $\max\beta \approx 116,54^\circ$. We note: $\max\beta = \max\gamma$, which is also evident from geometric considerations: four triangles with maximum γ form two rhombuses with a common edge BC; therefore, we have equality of angles: $\beta = ABH = GCD = \gamma$. $\triangleright$

4. On the Existence of an *RR*-Polyhedron Associated with a Snub Square Antiprism

In the work [12] the author found the angles of the rhombuses of the *rhombic* snub square antiprism, Fig. 2 d), which is connected with the regular facet $J85$ Johnson — snub square antiprism, Fig. 2 a).

Let us give a geometric justification for the existence of a *rhombic* snub square antiprism.

Theorem 2. *There is a non-composite RR-polyhedron of the second type with a symmetric 4-rhombic vertex and twenty-one regular faces.*

$\triangleleft$ As in [12], we remove four triangular faces, each of which is adjacent along the edge of the upper square face, and the square face of the polyhedron $J85$ itself. We obtain a polyhedral surface F with an boundary, a part of which is designated $BCDEF\ldots$ in the figure.

Recall that here and in what follows, when speaking about changing the angles of a polyhedral surface, we will mean the same changes in angles equivalent to those considered relative to the L axis. So the bendings of polyhedral surfaces with an boundary under consideration do not violate the symmetry of this surface relative to L, that is, they are S-bendings.

Let us prove that it is possible to attach a rhombic symmetric star with vertex V to the boundary $BCDEFF$ of surface F.

Let F_1 be the part of surface F that is square face $AMGH$ together with four triangles that have a common edge with this face, Fig. 2 c). Let F_1 be separated from surface S. Let us simultaneously increase the dihedral angles of triangular faces F_1 with the square face by the

same amount. In this case, angle KML and three other angles equivalent to it relative to the rotation-axis L of the 3rd order passing through the center of the square face will increase. When increasing the dihedral angles under consideration, a pair of triangles can be glued to angle KML and its equivalents: KDM, DLM, etc., Fig. 2 b). Such gluing is possible as long as the dihedral angle DM remains less than π. As a result of such gluing of eight triangles, we obtain a symmetrical surface F_2.

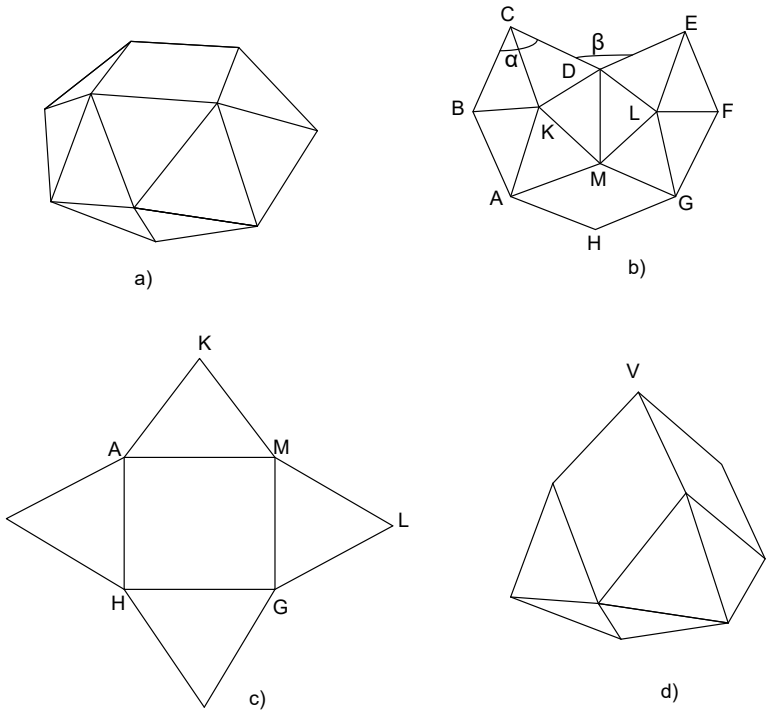

Fig. 2. a) — snub square antiprism; b)–c) — to the proof of the theorem 2; d) — rhombic snub square antiprism

Let us consider the vertex M of the surface F_2. By the Cauchy lemma applied to the 5-sided angle $MAKDLG$, we see that as the dihedral angles at the edges MA, MD, MG increase, the dihedral angles at the edges MK and ML must decrease. Due to symmetry, the angles at the edges KA and LG will also decrease. Then the plane angles BKD and DLF and their equivalents will decrease. We will glue pairs of triangles BCK and CKD into these angles, etc. We will obtain the surface F_3, symmetrically equivalent to F. At the last gluing, from the Cauchy lemma applied to the vertices K, L, etc., the dihedral angles at the edges BK and KD must increase, and the dihedral angle at the edge CK must decrease.

Thus, as a result of increasing the dihedral angle at edge MD, all three dihedral angles at vertex D increase, and then the plane angle CDE must also increase.

So, denoting α as the plane angle BCD and its equivalents, β as the angle CDE and its equivalents, ψ as the dihedral angle at edge MD and its equivalents, we obtain that with increasing angles ψ, angles β increase, and angles α decrease. It is also necessary to find the limits of change of angles α, β and ψ.

Earlier in [12] an equation (equation (43) in [12]) was obtained for calculating the acute angles α of a rhombic vertex V, which in the general case is not necessarily a rhombic vertex

looks like this:

$$\cos\beta = \cos^2\theta + \sin^2\theta\ \cos\left(2\arccos\left(\frac{1}{3}\frac{(1+2\sin\frac{\alpha}{2})\sqrt{3}}{\sqrt{3+4\sin\frac{\alpha}{2}-4\sin^2\frac{\alpha}{2}}}\right)\right.$$
$$\left. - 2\arccos\frac{\tan\frac{\theta}{2}}{\sqrt{3}} + \arccos\frac{-1+4\sin\frac{\alpha}{2}-4\sin^2\frac{\alpha}{2}}{3+4\sin\frac{\alpha}{2}-4\sin^2\frac{\alpha}{2}}\right), \quad (2)$$

here

$$\cos\theta = \frac{1}{4} + \frac{3}{4}\cos\left(\arccos\frac{\sin\frac{\alpha}{2}-1}{\sqrt{3}\cos\frac{\alpha}{2}} + \arccos\frac{\tan\frac{\alpha}{2}}{\sqrt{3}}\right).$$

In this equation, for the case where V is rhombic, $\cos\beta = \cos^2\frac{\alpha}{2}$. Equation (2) connects the angles α and β.

The values of the angle ψ are calculated using the relationship of this angle with the angle α. To find this relationship, note that $\psi = \varphi + \lambda$, where φ is the dihedral angle at the edge MD of the trihedral angle $MADC$ and λ is the acute angle of the plane of the triangle MKD with the plane of the trapezoid $ABDM$. For φ and λ, expressions (31) and (33) are obtained in [12], therefore for the angle ψ we have:

$$\psi = \arccos\frac{-1+4\sin\frac{\alpha}{2}-4\sin^2\frac{\alpha}{2}}{3+4\sin\frac{\alpha}{2}-4\sin^2\frac{\alpha}{2}} + \arccos\frac{1+2\sin\frac{\alpha}{2}}{\sqrt{3}\sqrt{3+4\sin\frac{\alpha}{2}-4\sin^2\frac{\alpha}{2}}}. \quad (3)$$

Note that we have also corrected a typo in [12]: in equation (31) in [12] α should be used instead of β.

The system of equations (2)–(3) connects all three angles α, β and ψ.

For convenience, we present a table of approximate values of the three angles α, β and ψ for the bending considered above:

Table 1

Values of angles in S-bending

α	β	ψ
118.16	60.00	164.26
114.75	73.10	166.50
92.90	111.87	180.00

For each value of α in degrees in the first column in the same row on the right, the corresponding values of β and ψ are presented. These values are obtained as follows.

For the Johnson polyhedron $J85$, obviously $\beta = 60°$. Using equation (2), we find the corresponding value of $\alpha \approx 118.16°$, and according to formula (3) — the value of $\psi \approx 164.26°$. The second row corresponds to the previously found values of α and β for the rhombic vertex, and according to formula (3) we find for the case of the rhombic vertex $\psi \approx 166.50°$. To obtain the values in the third row of the table, we take the limit value for the case of a convex polyhedron $\psi = 180°$ and, using formula (3), we find the value $\alpha \approx 92.90°$, and using formula (2) — the corresponding value $\beta = 111.87°$.

Thus, with the above-considered bending, $\min\alpha \approx 92.90°$ is achieved at $\max\beta \approx 111.87°$, and $\max\alpha \approx 118.16°$ is achieved at $\min\beta = 60°$.

So, using the S-bending of the surface F obtained from the polyhedron $J85$ by removing one square and four triangular faces adjacent along the edges to the square, with an increase

in the angles ψ we can obtain the values of the angles α and β for the case of a rhombic vertex, respectively equal to: $\approx 114.75^\circ$ and $\approx 73.10^\circ$. $\triangleright$

5. On the Existence of *RR*-Polyhedra Related to the Dodecahedron and the Cube

In the work [10] the author proved the existence of an *RR*-polyhedron of the first type with twenty-eight faces, which can be conditionally called a "rhombic augmented tetrahedron", which has four 3-rhombic obtuse-angled vertices, Fig. 3 a).

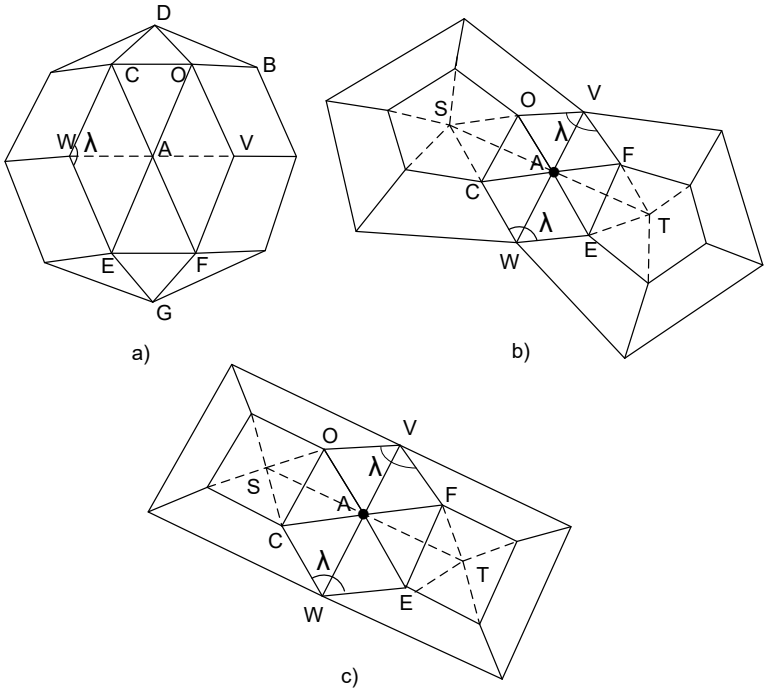

Fig. 3. a) — Rhombic augmented tetrahedron, b)–c) — to the proof of Theorems 3 and 4.

Here we prove the existence of a type II *RR*-polytope, which has 132 faces and 20 rhombic vertices.

Theorem 3. *There is a non-composite RR-polyhedron of the second type with twenty obtuse-angled rhombic vertices, having icosahedral symmetry.*

$\triangleleft$ Let us consider a regular dodecahedron with an edge of length 2. On each face of the dodecahedron we construct a regular pentagonal pyramid with edges of length 2. We cut off regular pyramids with unit edges from each pyramid. Thus, pentagonal regular truncated pyramids with unit edges of the upper and lateral faces will be built on the faces of the dodecahedron. Two of the truncated pyramids with a common edge WV are shown in Fig. 3 b). Thus, WV is one of the edges of the non-convex polyhedron M obtained by the above construction.

Consider a non-convex dihedral angle with edge WA. For a trihedral angle $WCAE$ we have the relation:

$$\cos\lambda = \cos\frac{\pi}{3}\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\sin\frac{\pi}{3}\cos\widehat{\phi}, \tag{4}$$

where λ is the plane angle CWE (and the equal angle OVF), $\widehat{\phi}$ is the dihedral angle with edge WV. The angle $\widehat{\phi}$ is the complement to 2π of the sum of two dihedral angles at the

base of a regular pentagonal pyramid α and the dihedral angle of a dodecahedron β. Since $\alpha \approx 37.377°$, and $\beta \approx 116.565°$, then $\widehat{\phi} \approx 168.680°$. From here, using (4), we get: $\lambda \approx 119.04°$.

Let point A' be the midpoint of edge WV. Rotate equilateral triangle $CA'O$ around side CO so that its vertex A' is located in the midpoint of segment ST, where S and T are the points where the vertices of the pentagonal pyramids were located before they were cut off. We denote the new position of point A' by A. Thus, A' is the projection of point A onto edge WV. We similarly rotate triangle $EA'F$. Due to symmetry, point A will be common to both triangles.

Since CO and EF were the midlines of the pentagonal pyramids with vertices S and T, the quadrangles $WCAE$ and $VOAF$ are *flat* and equilateral, that is, rhombuses with obtuse angles $\lambda \approx 119.04°$.

Carrying out similar constructions for each two adjacent truncated pyramids of the polyhedron M, we obtain a convex RR-polyhedron M' of the second type, the faces of which are sixty rhombi, twelve regular pentagons and sixty regular triangles. At each of the twenty vertices of the polyhedron M', which were the vertices of a dodecahedron before the construction, three rhombi meet with their obtuse angles. ▷

Similar to the previous theorem, the existence of an RR-polyhedron of the second type, which has 54 faces and 8 rhombic vertices, is proved.

Theorem 4. *There is a non-composite RR-polyhedron of the second type with eight obtuse-angled rhombic vertices, having octahedral symmetry.*

◁ Let us dwell only on the differences from the proof of the previous theorem. Instead of a dodecahedron, here we consider a cube with an edge of 2 and construct regular equilateral quadrangular pyramids on its faces. By cutting off these pyramids with unit edges, we obtain a non-convex polyhedron M. Completing M by adding rhombic faces, as in the previous theorem, we obtain a convex RR-polyhedron with fifty-four faces. Further, we indicate here only the obtuse angles of the rhombi, which we find in the same way as in the case of the dodecahedron.

$\widehat{\phi}$ — dihedral angle with edge WV, Fig. 3 c). The last angle is found as the complement to 2π of the sum of two dihedral angles at the base of a regular quadrangular pyramid, equal in $\approx 54.74°$, and the dihedral angle of the cube. Thus, $\widehat{\phi} \approx 160.528°$. From here, using formula (4), we obtain: $\lambda \approx 117.20°$ — obtuse angles of the rhombus. ▷

Remark. In the case of the octahedron and icosahedron, constructions similar to those of the dodecahedron and cube will not lead to new polyhedra. Indeed, in the case of the octahedron, the angles of the rhombi will be right angles and, therefore, four rhombi will not form a rhombic star; in the case of the icosahedron, the rhombic stars will be concave and the polyhedron will not be convex.

7. Conclusion

After a complete enumeration of composite RR-polyhedra of the second type, the question of the existence of non-composite such polyhedra naturally arose. This paper proves the existence of four non-composite RR-polyhedra with regular faces of different types and without conditional edges; three of these polyhedra have obtuse rhombic vertices: a 31-sided polyhedron, a 54-sided polyhedron, and a 132-sided polyhedron. Thus, if we do not restrict ourselves to rhombic faces composed of two regular triangles, then the list of polyhedra with rhombic vertices and conditional edges can be extended by adding the four found non-composite polyhedra without conditional edges.

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Vladimir I. Subbotin
Platov South Russian State Polytechnic University (NPI),
132 Prosveshheniya Ave., Novocherkassk 346400, Russia,
Associate Professor of the Department of Applied Mathematics
E-mail: geometry@mail.ru
https://orcid.org/0000-0003-4204-8195

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О ДОКАЗАТЕЛЬСТВАХ СУЩЕСТВОВАНИЯ НЕСОСТАВНЫХ МНОГОГРАННИКОВ С РОМБИЧЕСКИМИ ВЕРШИНАМИ

Субботин В. И.[1]

[1] Южно-Российский государственный политехнический университет (НПИ) им. М. И. Платова, Россия, 346400, Новочеркасск, ул. Просвещения, 132

E-mail: geometry@mail.ru

Аннотация. Ранее автором рассматривались замкнутые выпуклые многогранники в E^3 с симметричными ромбическими вершинами и правильными гранями (так называемые *RR*-многогранники) в двух случаях: 1) правильные грани у таких многогранников одного типа, 2) правильные грани различного типа, но многогранники являются составными. Из ранее опубликованных работ автора следует, что

в обоих случаях список RR-многогранников является полным. В первом случае их оказалось двадцать четыре, а во втором — пятьдесят четыре. Вершина V многогранника называется ромбической, если звезда $\mathrm{Star}(V)$ из граней вершины V многогранника состоит из n равных и одинаково расположенных ромбов (не квадратов), имеющих общей вершиной V. Если вершина V лежит на оси вращения порядка n звезды $\mathrm{Star}(V)$, то V называется симметричной. Симметричная ромбическая вершина V называется тупоугольной, если ромбы звезды $\mathrm{Star}(V)$ в вершине V сходятся своими тупыми углами. Звезда $\mathrm{Star}(V)$ называется при этом симметричной ромбической. Грани RR-многогранника можно разбить на два непустых непересекающихся множества: множество равных симметричных ромбических звезд, не имеющих общих ребер, и множество правильных граней. Примером RR-многогранника является удлиненный ромбододекаэдр. Если в RR-многограннике правильные грани одного типа, то такой RR-многогранник отнесен к первому типу. Частными случаями RR-многогранников являются некоторые из правильногранников с условными ребрами, например удлиненный ромбододекаэдр и дважды дополненный кубооктаэдр. При этом ромбические грани правильногранных многогранников с условными ребрами в отличие от ромбов RR-многогранников, составлены только из правильных треугольников. В настоящей работе даны конструктивные доказательства существования четырех несоставных RR-многогранников. Для двух из них (25-гранника и 31-гранника) ранее автором были найдены углы ромбов, образующих симметричные гранные звезды вершин. В случае 25-гранника ромбическая вершина имеет остроугольную гранную звезду, т. е. ромбы в ней сходятся своими острыми углами, а в случае 31-гранника гранная звезда тупоугольная. Существование двух других несоставных RR-многогранников (132-гранника и 54-гранника) доказано на основе правильного додекаэдра и куба.

Ключевые слова: несоставной RR-многогранник, гранная звезда вершины, симметричная ромбическая вершина.

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